Field of homogeneous Plane in Quantum Electrodynamics

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Abstract

We study quantum electrodynamics coupled to the matter field on singular background, which we call defect. For defect on the infinite plane we calculated the fermion propagator and mean electromagnetic field. We show that at large distances from the defect plane, the electromagnetic field is constant what is in agreement with the classical results. The quantum corrections determining the field near the plane are calculated in the leading order of perturbation theory.

1 Introduction

It is a long-standing problem to formulate a quantum gauge theory with boundaries [1, 2, 3]. There are different approaches for its solution. One can impose boundary condition on the gauge and ghost fields [4]. Other possibility in quantum electrodynamics (QED) is to put "physical" boundary conditions on the electromagnetic field strength and use either explicitly gauge-invariant methods (see for example [5]), or consider these boundary condition as constrains quantizing the potential A_μ. From the formal point of view, shortcoming of the models of such a kind is that the gauge invariance, renormalizability or locality appear to be broken. There is another reason to doubt that these models describe correctly the physics of interaction of quantum field with the boundary, since the same sharp condition is imposed on all the modes of the fields. In reality the modes of the field with high enough frequencies are not constrained by the matter and are not influenced by its presence. We restrict ourself with these short comments about the

method of quantization with boundary constrains on the fields because it is not used in this paper[1].

The other approach which we follow here is to model boundary effects with a static background (defect) coupled to the quantum fields. The simplest background is the singular one. The first attempt to build renormalizable local QFT with boundaries in this way has been made by Symanzik in [7]. It was shown that some simple boundary conditions (namely Dirichlet and Neumann ones) are implemented by adding a singular interaction to the action density. The technique was generalized to manifolds with curved boundaries in subsequent work [8]. Recently the models of the scalar field theory in 2- and 3-dimensions were considered in [9, 10, 11] (in the later paper structure of divergencies and corresponding renormalization procedure are also discussed). The method was successfully applied for the investigation of the critical behavior of magnets and alloys with free surfaces [12, 13]. In context of the Casimir problem the models of interaction of the free quantum field with singular background (δ function-like potential) were considered by many authors [14, 15, 16, 17, 18, 19]. Many important results are known also for non-relativistic quantum mechanic systems with δ-potential [20, 21].

The problem of renormalization of QED with singular background ware considered in [22, 23]. The radiative QED correction to the Casimir energy was calculated in [24]. A review on Casimir effect was published recently [25], where most results and controversies in the field are discussed.

Theory of Casimir effect based on assumption that it is a macroscopic phenomena generating by vacuum fluctuations of QED fields is in a good agreement with experimental data [26, 27, 28]. It is naturally to expect that the Casimir effect is not unique macro-manifestation of quantum fields' fluctuations, and in many situations the description of macro-system behavior obtained by classical electrodynamics (ED) needs essential quantum corrections. The calculation of them is a theoretical task being of practical importance for development of nano-technologies and design of microdevices [29]. A possible method of its solution is proposed in this paper.

A typical ED statement of problem is to find the electric and magnetic fields for given boundary conditions, charge and current distributions [30, 31]. In this paper we consider the QED version of simplest problem of such a kind. We study the gauge invariant, local, renormalizable model of simple defect on the plane in QED. It is suggested for calculation of quantum corrections for the fields of a plane in classical electrodynamics. It is essential that for renormalizability of the model the direct interactions of the boundary both with the photon and with the Dirac's fields are necessary. We calculate the leading order approximations for mean strengths of electric and magnetic fields expressed by usual relations in terms of components of electromagnetic field tensor. The results look like classical ones on the large distance from the plane. For small distances $r \to 0$ the strength of fields appear to be singular as $C_1/r^2 + C_2/r$, where C_1, C_2 are constants. It is essentially non-classical effect generated by interaction of the Dirac fields with defect

[1] Many results of application of this method for investigation of Casimir effect are presented in [6, 25]

on the plane. In the framework of the free scalar field theory with singular background a similar phenomena was studied in [32].

2 Statement of the problem

We consider the QED with homogeneous defect on the infinite plane $x_3 = 0$ invariant in respect to coordinate reflections. It is specified by the action functional of the form

$$S_{def}(\bar{\psi}, \psi, A; \lambda, q, l) = S_{\lambda q}(\bar{\psi}, \psi) + S_l(A), \tag{1}$$

$$S_{\lambda q}(\bar{\psi}, \psi) \equiv \int \bar{\psi}(\vec{x}, 0)(\lambda + \hat{q})\psi(\vec{x}, 0)d\vec{x}, \quad S_l(A) \equiv \int lA(\vec{x}, 0)d\vec{x} + \int l' \partial_3^2 A(\vec{x}, 0)d\vec{x},$$

where $\bar{\psi}$, ψ are the Dirac's spinor fields, A is the electromagnetic vector-potential, q, l, l' are fixed 4-vectors, $\hat{q} = q_\mu \gamma^\mu$ (γ^μ are the Dirac's gamma-matrices), and we used the short hand notations for the 4-vector x: $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$. The notations of this kind will also be used later. For gauge invariance of the defect action it is necessary to set $l_3 = l'_3 = 0$ [2]. With this restriction the functional (1) is a most general form of local defect action without parameters of negative dimension and being invariant in respect to coordinate reflections and gauge transformations. Therefore in virtue of usual criteria of renormalization theory the model defined by addition to the QED action the defect action (1) is renormalizable [7]. It remains to be renormalizable for $l' = 0$ too, because as we see below, there are no divergencies which need for their cancellation the l'-term in the S_l term.

The physical meaning of the vector $l = (\vec{l}, 0)$ is very simple. It defines the classical 4-current on the plane defect. By neglecting in our model the interaction of the photon and Dirac fields, the mean electromagnetic field coincides with solution of Maxwell equations with the current (supported on the plane) defined by S_l. Vector q and scalar λ describe the interaction of current and density of Dirac field with material defect. Interaction of vacuum fluctuations of the Dirac field with the background generates quantum corrections to usual classical effects.

We calculate the leading approximation for electromagnetic field generated by the defect. Only the l-term in S_l appears to be necessary for cancellation of ultra-violet divergences in our results. We set $l' = 0$ and do not consider a trivial contribution to the first order effects from l'-term in S_l. We choose the vector $\vec{l}$ proportional to $\vec{q}$: $\vec{l} = \vec{q}\xi$, and show that this "minimal" form of S_l with only one extra to S_f parameter ξ provides the cancellation of divergencies by renormalization of ξ.

The full action of our model has the form

$$S(\bar{\psi}, \psi, A) = S_{QED}(\bar{\psi}, \psi, A) + S_{def}(\bar{\psi}, \psi, A; \lambda, q, \xi), \tag{2}$$

[2]We consider the gauge transformations $A_\mu \to A_\mu + \partial_\mu \phi$ not changing asymptotic of the field $A_\mu(x)$ for large x. It follows from this assumption that $\lim_{x_i \to \pm\infty} \phi(x) = \phi_0$, where ϕ_0 is a constant being the same for all $i = 0, 1, 2, 3$. Therefore $S_l(A)$ is gauge invariant.

where $S_{QED}(\bar{\psi}, \psi, A)$ is the usual QED action:

$$S_{QED}(\bar{\psi}, \psi, A) = \int \bar{\psi}(x)(i\hat{\partial} - e\hat{A}(x) - m)\psi(x)dx - \frac{1}{4}\int F_{\mu\nu}(x)F^{\mu\nu}(x)dx.$$

Here,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \hat{A} = A_\mu \gamma^\mu,$$

A_μ is the potential of electromagnetic fields and gamma matrices fulfill the commutation relations

$$\{\gamma_\mu, \gamma_\nu\} \equiv \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$$

with metric tensor $g_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$. The considered model is invariant in respect to translations and Lorenz transformations leaving unchanged the coordinate x_3, if the vectors $\vec{q}$ transformed correspondingly.

In virtue of Lorenz-invariance of the model in the sense mentioned above, we can classify the defect properties with the value of the invariant $\vec{q}^2 \equiv q_0^2 - q_1^2 - q_2^2$. So, we have 3 cases: $\vec{q}^2 = \kappa^2 > 0$, $\vec{q}^2 = -\kappa^2 < 0$ and $\vec{q}^2 = 0$, and we can choose (and restrict ourselves with) three coordinate systems where $\vec{q} = (\kappa, 0, 0)$, $\vec{q} = (0, \kappa, 0)$ $\vec{q} = (\kappa, \kappa, 0)$ accordingly.

This means that interaction of QED fields with the defect is described by four parameters: κ, $\tau \equiv q_3$, λ and ξ.

In this paper we calculate the mean tensor $F_{\mu\nu}$ of electromagnetic field:

$$\mathcal{F}_{\mu\nu} = C \int F_{\mu\nu} e^{iS(\bar{\psi},\psi,A)} DAD\bar{\psi}D\psi, \tag{3}$$

where the constant C is defined as follows

$$C^{-1} = \int e^{iS(\bar{\psi},\psi,A)} DAD\bar{\psi}D\psi.$$

For calculations, it is more convenient to use the Euclidean version of the model. It is obtained by the usual Wick rotation

$$t \to -it, \quad A_0 \to iA_0, \quad \gamma_0 \to i\gamma_0, \quad q_0 \to iq_0.$$

Then the Euclidean action can be written as

$$S^E = S^E_{QED} + S^E_{def},$$

where

$$S^E_{QED} = \bar{\psi}(i\hat{\partial} - e\hat{A} + m)\psi + \frac{1}{4}F_{\mu\nu}F_{\mu\nu}, \quad S^E_{def}(\bar{\psi}, \psi, A; \lambda, q, \xi) = \bar{\Psi}(\lambda + \hat{q})\Psi + \xi(\vec{q}\vec{A}),$$

$$\bar{\Psi}(\vec{x}) = \bar{\psi}(\vec{x}, 0), \qquad \Psi(\vec{x}) = \psi(\vec{x}, 0).$$

The matrix $\hat{a}$ corresponding to 4-vector a is defined in Euclidean theory as $\hat{a} = \sum_{\mu=0}^{3} a_\mu\gamma_\mu$. For convenience of calculations we shall use the projection operator onto the plane $x_3 = 0$

$$\Omega(\vec{x}, y) = \delta(\vec{x} - \vec{y})\delta(y_3) \tag{4}$$

and present the function $\bar{\Psi}(\vec{x})$, $\Psi(\vec{x})$ in the form

$$\bar{\Psi}(\vec{x}) = \int \bar{\psi}(y)\Omega^T(y,\vec{x})dy \equiv \bar{\psi}\Omega^T, \quad \Psi(\vec{x}) = \int \Omega(\vec{x},y)\psi(y)dy \equiv \Omega\psi.$$

The tensor $F_{\mu\nu}$ is gauge invariant, therefore $\mathcal{F}_{\mu\nu}$ is independent of the choose of the gauge. We provide calculations in the Feynman's gauge[3], using the formula

$$D_{\mu\nu}(x,y) = \frac{\delta_{\mu\nu}}{4\pi^2(x-y)^2}$$

for the photon propagator in configuration space. Integrating by parts in functional integral (3) we obtain

$$\mathcal{F}_{\mu\nu}(x) = \frac{1}{2\pi^2}\int d^4y \frac{(x_\nu - y_\nu)J_\mu(y) - (x_\mu - y_\mu)J_\nu(y)}{(x-y)^4}, \tag{5}$$

where

$$J_\mu(y) = ej_\mu(y) - l_\mu\delta(y_3), \;\; j_\mu(y) = C\int e^{-S(\bar{\psi},\psi,\vec{q})}\bar{\psi}(y)\gamma_\mu\psi(y)D\bar{\psi}D\psi. \tag{6}$$

In virtue of invariance of the action (2) in respect to translation of coordinates x_0, x_1, x_2 and reflection of x_3, $j_\mu(y)$ is an even function of coordinate y_3 only: $J(y) = J(y_3) = J(-y_3)$, and after integration over $\vec{y}$ in (5) we obtain the following result

$$\mathcal{F}_{\mu\nu}(x) = \frac{1}{2}\int_{-x_3}^{x_3}[\delta_{\nu3}J_\mu(y_3) - \delta_{\mu3}J_\nu(y_3)]dy_3 = \tag{7}$$

$$= \mathrm{Sign}(x_3)\left\{e\int_0^{|x_3|}[\delta_{\nu3}j_\mu(y_3) - \delta_{\mu3}j_\nu(y_3)]dy_3 - \xi(\delta_{\nu3}q_\mu - \delta_{\mu3}q_\nu)\right\},$$

where $\mathrm{Sign}(x_3)$ is the signum-function: $\mathrm{Sign}(x_3) = x_3/|x_3|$.

The vector j_μ in (6) is the current generated by vacuum fluctuations of Dirac fields. In virtue of Farri's theorem, it vanishes in absence of defect. By usual methods of renormalization theory and its modification for the quantum field theory with singular background [7] one can prove that in the framework of renormalized perturbation theory $j_\mu(y_3)$ is presented by a sum of diagrams with all the necessary subtractions. Therefore it is finite (for the leading approximation it will be clear from the evident formula below), and in calculation of (7) there is the only problem - one with non-integrable singularity of $j_\mu(y_3)$ at $y_3 = 0$. Therefore the integral (7) needs a regularization. Since for $x_3 \neq 0$ the derivative of $\mathcal{F}_{\mu\nu}(x_3)$ in respect to x_3 is finite, we obtain the finite value of $\mathcal{F}_{\mu\nu}(x_3)$ subtracting from integral in the left hand of (7) a constant dependent on the chosen regularization. This subtraction can be generated by term S_l with appropriate choice of the parameter ξ. Therefore the l'-term in S_l is not necessary for renormalizability of considered model.

[3]This gauge can be fixed by the usual Faddeev-Popov trick

The leading approximation $j_\mu^{(0)}$ for j_μ can be presented as

$$j_\mu^{(0)} \equiv Tr(\hat{S}(y,y)\gamma_\mu). \tag{8}$$

Here $\hat{S}(x,y)$ is the free fermion propagator in the theory with defect $S_{\lambda q}(\bar{\psi},\psi)$

$$\hat{S}_{\alpha\beta}(x,y) = C_f \int e^{-S_f(\bar{\psi},\psi,q)}\bar{\psi}_\beta(y)\psi_\alpha(x) D\bar{\psi}D\psi. \tag{9}$$

We used in (9) the notation $S_f(\bar{\psi},\psi,q)$ for the fermion part of the action S^E

$$S_f(\bar{\psi},\psi,q) = S^E(\bar{\psi},\psi,A;\lambda,q,\xi)|_{A=0} = \bar{\psi}\hat{K}\psi + \bar{\Psi}(\lambda+\hat{q})\Psi,\ \hat{K}\equiv i\hat{\partial}+m,$$

and

$$C_f^{-1} = \int e^{-S_f(\bar{\psi},\psi,q)} D\bar{\psi}D\psi.$$

We restrict ourselves with calculation of the main approximation for $\mathcal{F}_{\mu\nu}$, and in virtue of (8), the most nontrivial part of this problem is to find an evident form for the fermion propagator $\hat{S}$ defined by (9).

3 Calculation of $\hat{S}$

The right hand side of (9) can be obtained by differentiation of generating functional

$$G(\bar{\eta},\eta) \equiv C_f \int \exp\left\{-S_f(\bar{\psi},\psi;q) + \bar{\psi}\eta + \bar{\eta}\psi\right\} \mathcal{D}\bar{\psi}\mathcal{D}\psi$$

where $\bar{\eta}$, η are the fermion sources. Obviously,

$$\hat{S}_{\alpha\beta}(x,y) = \frac{\delta}{\delta\bar{\eta}_\alpha(x)}\frac{\delta}{\delta\eta_\beta(y)} G(\bar{\eta},\eta)|_{\bar{\eta},\eta=0},\ G(\eta,\bar{\eta}) = \exp\left\{\bar{\eta}\hat{S}\eta\right\}.$$

To calculate $G(\bar{\eta},\eta)$ we present the contribution of the defect in the following form

$$\exp\left\{-\bar{\Psi}(\lambda+\hat{q})\Psi\right\} = c\int \exp\left\{\bar{\zeta}\zeta + \bar{\zeta}(\lambda+\hat{q})\Psi + \bar{\Psi}\zeta\right\}\mathcal{D}\bar{\zeta}\mathcal{D}\zeta,\ c^{-1} = \int \exp\left\{\bar{\zeta}\zeta\right\}\mathcal{D}\bar{\zeta}\mathcal{D}\zeta,$$

and write $G(\bar{\eta},\eta)$ as

$$G(\bar{\eta},\eta) = cC_f \int \exp\left\{-\bar{\psi}\hat{K}\psi + \bar{\zeta}\zeta + \bar{\psi}\eta + \bar{\eta}\psi + \bar{\Psi}\zeta + \bar{\zeta}(\lambda+\hat{q})\Psi\right\}\mathcal{D}\bar{\psi}\mathcal{D}\psi D\bar{\zeta}D\zeta$$

$$= cC_f \int \exp\left\{-\bar{\psi}\hat{K}\psi + \bar{\zeta}\zeta + \bar{\psi}\theta + \bar{\theta}\psi\right\}\mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\bar{\zeta}\mathcal{D}\zeta,$$

where

$$\bar{\theta} \equiv \bar{\eta} + \bar{\zeta}(\lambda+\hat{q})\Omega, \quad \theta \equiv \eta + \Omega^T\zeta.$$

Now, integrating over the fields $\bar{\psi}$, ψ and then over ζ, $\bar{\zeta}$ we obtain

$$G(\eta, \bar{\eta}) = \exp\left\{\bar{\eta}\hat{S}\eta\right\},$$

where

$$\hat{S} = \hat{D} + \hat{S}_{def}, \qquad \hat{S}_{def} = -\hat{D}\Omega^T\hat{Q}^{-1}(\lambda + \hat{q})\Omega\hat{D}. \tag{10}$$

We used here the following notations

$$\hat{D} \equiv \hat{K}^{-1}, \;\; \hat{Q} \equiv \mathbf{1} + (\lambda + \hat{q})\Omega\hat{D}\Omega^T.$$

We see that the propagator $\hat{S}$ in our model is the sum of propagator $\hat{D}$ of usual QED and $\hat{S}_{def}$ generated by the defect. The propagator $\hat{D}$ does not make contribution into (8), and we can present $j_\mu^{(0)}$ as

$$j_\mu^{(0)}(y) = Tr(\hat{S}_{def}(y,y)\gamma_\mu). \tag{11}$$

Now, we calculate $\hat{S}_{def}$ in an evident form, which will be used to obtain the final result for $\mathcal{F}_{\mu\nu}$ in considered approximation. In virtue of translation invariance for coordinates $\vec{x}$, it is natural in our model to use for calculations the 3-dimensional Fourier transformation defined as follows

$$F(\vec{x}, x_3) = \frac{1}{(2\pi)^3}\int d\vec{p}\, e^{i\vec{p}\vec{x}}F(\vec{p}, x_3).$$

For the propagator $\hat{D}$ we have

$$\hat{D}(x) \equiv \hat{D}(\vec{x}, x_3) = \frac{1}{(2\pi)^3}\int \frac{m + \hat{\mathbf{p}} + iE\gamma_3 \mathrm{Sign}(x_3)}{2E} \exp\left\{-E|x_3| + i\vec{p}\vec{x}\right\} d\vec{p},$$

where

$$\hat{\mathbf{p}} \equiv \vec{p}\vec{\gamma}, \quad E \equiv \sqrt{\vec{p}^2 + m^2}.$$

Hence,

$$\hat{D}(\vec{p}, x_3) = \frac{m + \hat{\mathbf{p}} + iE\gamma_3 \mathrm{Sign}(x_3)}{2E}\, e^{-E|x_3|}.$$

For calculation of $\hat{S}_{def}(x,y)$ we note that in virtue of (4)

$$\Omega\hat{D}\Omega^T(\vec{p}) = \hat{D}(\vec{p}, 0) = \frac{m + \hat{\mathbf{p}}}{2E}.$$

So, we can write

$$\hat{S}_{def}(x,y) = \frac{1}{(2\pi)^3}\int d\vec{p}\; e^{i\vec{p}(\vec{x}-\vec{y})}\hat{S}_{def}(\vec{p}; x_3, y_3), \tag{12}$$

where

$$\begin{aligned}\hat{S}_{def}(\vec{p}; x_3, y_3) &\equiv -\hat{D}(\vec{p}; x_3)\left(1 + (\lambda + \hat{q})\frac{m + \hat{\mathbf{p}}}{2E}\right)^{-1}(\lambda + \hat{q})\hat{D}(\vec{p}; -y_3) \\ &= -\frac{[m + \hat{p}_1][M + \hat{r}][m + \hat{p}_2]}{2E^2[E\,(4 + q^2 + \lambda^2) + 4\,(\lambda m - \vec{p}\vec{q})]}\, e^{-E(|x_3| + |y_3|)}.\end{aligned} \tag{13}$$

Here, we have denoted

$$p_1 = (\vec{p}, i\mathrm{Sign}(x_3)E), \quad p_2 = (\vec{p}, -i\mathrm{Sign}(y_3)E),$$
$$M = 2E\lambda + (\lambda^2 + q^2)m, \quad r = \left(2E\vec{q} - (\lambda^2 + q^2)\vec{p}, 2Eq_3\right).$$

The Dirac field propagator $\hat{S}$ for the defect with $\lambda = q_1 = q_2 = q_3 = 0$ was calculated in [15, 16], and our result (13) generalize one obtained there, and coincides with it for particular values of defect parameters.

Now, we use (13) for calculation of electromagnetic field generated by the defect.

4 Calculations of $\mathcal{F}_{\mu\nu}$

Setting the right hand side of (12) into (11) and using (13), one obtains

$$j^{(0)}(x) = (\vec{j}^{(0)}(x), j_3^{(0)}(x)) = \frac{e}{(2\pi)^3}\int d\vec{p}\, Tr[(\vec{\gamma}, \gamma_3)S_{def}(\vec{p}, x_3, x_3)]$$

$$= -\frac{e}{\pi^3}\int d\vec{p} \frac{e^{-2E|x_3|}}{E(E\,(4 + q^2 + \lambda^2) + 4\,(\lambda m - \vec{p}\vec{q}))}\left(\vec{p}\,(\vec{p}\vec{q} - m\lambda) - E^2\vec{q},\ 0\right).$$

After integration over angular variables $j^{(0)}(x)$ is written in the form

$$j^{(0)}(x) = e\,(\vec{q}, 0)\,\phi(x_3),$$

$$\phi(x_3) \equiv \frac{1}{2\pi^2|\vec{q}|\alpha^2}\int_0^\infty p\, e^{-2E|x_3|}\left[2p\alpha - (E(1-\alpha^2) + m\beta))\,\ln\frac{E + \alpha p + \beta m}{E - \alpha p + \beta m}\right] dp\,, \quad (14)$$

where

$$\alpha = \frac{4|\vec{q}|}{4 + q^2 + \lambda^2}, \quad \beta = \frac{4\lambda}{4 + q^2 + \lambda^2}. \quad (15)$$

For $x_3 \neq 0$ the current $j^{(0)}(x_3)$ is finite since the integral (14) converges. However we can not set $j(y_3) = j^{(0)}(y_3)$ in (7) directly, since $j^{(0)}(y_3)$ has a non-integrable singularity at $y_3 = 0$. The regularized form of the leading approximation of $\mathcal{F}_{\mu\nu}$ can be obtained by introducing a cut-off parameter Λ in (14) and substituting then of regularized current $j^{(0)}(\Lambda)$ into (7). In doing so one obtains the following result

$$\mathcal{F}_{\mu\nu}(x) = e(q_\mu\delta_{\nu 3} - q_\nu\delta_{3\mu})\mathrm{Sign}(x_3)\left[F(x_3, \Lambda) - \frac{\xi}{2}\right],$$

where

$$F(x_3, \Lambda) \equiv \frac{1}{4\pi^2|\vec{q}|\alpha^2}\int_0^\Lambda dp\frac{\left(1 - e^{-2E|x_3|}\right)p}{E}\left[2p\alpha - (E(1-\alpha^2) + m\beta)\,\ln\frac{E + \alpha p + \beta m}{E - \alpha p + \beta m}\right].$$

The limit $\lim_{x\to\infty} F(x,\Lambda) \equiv F(\infty,\Lambda)$ can be calculated in an evident form

$$F(\infty,\Lambda) = \frac{1}{4\pi^2|\vec{q}|\alpha^2}\Bigg\{\alpha\Lambda(\sqrt{\Lambda^2+m^2}+\beta m) + m^2[\xi(\Lambda,a_1,b_1)+\xi(\Lambda,a_2,b_2)]$$

$$+\left(\frac{(\Lambda^2+m^2)(1-\alpha^2)}{2}+\beta m\sqrt{\Lambda^2+m^2}\right)\ln\left[\frac{\sqrt{\Lambda^2+m^2}-\alpha\Lambda+\beta m}{\sqrt{\Lambda^2+m^2}+\alpha\Lambda+\beta m}\right]\Bigg\}.$$

Here

$$\xi(\Lambda,a,b) = \frac{b}{\sqrt{a^2-1}}\ln\left[\frac{a\sqrt{\Lambda^2+m^2}-m+\Lambda\sqrt{a^2-1}}{a\,m-\sqrt{\Lambda^2+m^2}}\right],$$

and

$$a_1 = -\frac{\beta+\alpha\sqrt{\alpha^2+\beta^2-1}}{1-\alpha^2}, \quad a_2 = -\frac{\beta-\alpha\sqrt{\alpha^2+\beta^2-1}}{1-\alpha^2},$$

$$b_1 = \frac{(\alpha^2+\beta^2)(\alpha\beta+\sqrt{\alpha^2+\beta^2-1})}{2(1-\alpha^2)}, \quad b_2 = \frac{(\alpha^2+\beta^2)(\alpha\beta-\sqrt{\alpha^2+\beta^2-1})}{2(1-\alpha^2)} \tag{16}$$

The asymptotic of $F(\rho,\Lambda)$ for large Λ can be written as

$$F(\rho,\Lambda) = c_2\frac{\Lambda^2}{m^2}+c_1\frac{\Lambda}{m}+c_0+f(\rho)+O\left(\frac{m^2}{\Lambda^2}\right),$$

where

$$f(\rho) \equiv -\frac{1}{4\pi^2|\vec{q}|\alpha^2}\int_0^\infty dp\frac{e^{-2E|\rho|}p}{E}\left[2p\alpha-(E(1-\alpha^2)+m\beta)\,\ln\frac{E+\alpha p+\beta m}{E-\alpha p+\beta m}\right], \tag{17}$$

c_0, c_1, c_2 are the constants:

$$c_2 = \frac{m^2}{8\pi^2|\vec{q}|\alpha^2}\left\{2\alpha+(\alpha^2-1)\ln\frac{1+\alpha}{1-\alpha}\right\}, \; c_1 = \frac{\beta m^2}{4\pi^2|\vec{q}|\alpha^2}\left\{2\alpha-\ln\frac{1+\alpha}{1-\alpha}\right\},$$

$$c_0 = c_2+\frac{m^2}{4\pi^2|\vec{q}|\alpha^2}\left\{\frac{\alpha\beta^2}{(1-\alpha^2)}+\frac{b_1\ln[-a_1-\sqrt{a_1^2-1}]}{\sqrt{a_1^2-1}}+\frac{b_2\ln[-a_2-\sqrt{a_2^2-1}]}{\sqrt{a_2^2-1}}\right\}$$

where we used the notations (15),(16), and $O\left(\frac{m^2}{\Lambda^2}\right)\to 0$ for $\Lambda\to\infty$. Requirement of $\mathcal{F}_{\mu\nu}$ being finite for $\Lambda\to\infty$ means that ξ depends on Λ, and for large Λ its asymptotic has the form

$$\xi(\Lambda) = 2\left(c_2\frac{\Lambda^2}{m^2}+c_1\frac{\Lambda}{m}\right)+\chi+O\left(\frac{m^2}{\Lambda^2}\right).$$

Here the parameter χ is the renormalized value of ξ .

Thus, if we denote $\vec{\mathcal{F}} = (\mathcal{F}_{03},\mathcal{F}_{13},\mathcal{F}_{23})$, then for $\Lambda\to\infty$ we obtain the following result

$$\vec{\mathcal{F}}(x) = e\,\mathrm{Sign}(x_3)\,\vec{q}\left(c_0+f(x_3)-\frac{\chi}{2}\right).$$

The asymptotics of function $f(\rho)$ (17) are of the form

$$f(\rho) \underset{\rho\to\infty}{=} -\frac{\alpha m^2 e^{-2m|\rho|}}{8|\hat{q}|(\pi m|\rho|)^{3/2}(1+\beta)}\,(1+O(1/\rho)),$$

$$f(\rho) \underset{\rho\to 0}{=} -\frac{c_2}{2m^2\rho^2} - \frac{c_1}{2m|\rho|} - c_0 + c_2 + O(\rho).$$

Hence

$$\vec{\mathcal{F}}(x) \underset{x_3\to\infty}{=} e\,\mathrm{Sign}(x_3)\,\vec{q}\left(c_0 - \frac{\chi}{2} - \frac{\alpha m^2 e^{-2m|x_3|}}{8|\hat{q}|(\pi m|x_3|)^{3/2}(1+\beta)}\,(1+O(1/x_3))\right),$$

$$\vec{\mathcal{F}}(x) \underset{x_3\to\ 0}{=} -e\,\mathrm{Sign}(x_3)\,\vec{q}\left(\frac{c_2}{2m^2x_3^2} + \frac{c_1}{2m|x_3|} + \frac{\chi}{2} - c_2 + O(x_3)\right).$$

We see, that on large distances the field generated by defect is of the same form as the field of the plane in classical ED [30, 31], and on the small distance x_3 it has non-classical behavior $\vec{\mathcal{F}} \sim e\vec{q}c_2/2m^2x_3^2$. Let us consider 3 cases: 1) $q = (\kappa,0,0,\tau) \equiv q^{(1)}$; 2) $q = (0,\kappa,0,\tau) \equiv q^{(2)}$; 3) $q = (\kappa,\kappa,0,\tau) \equiv q^{(3)}$. If $q = q^{(1)}$, the defect generates pure electric field: $H_1 = H_2 = H_3 = 0,\ E_1 = E_2 = 0,\ E_3 = -i\mathcal{F}_{03}(-i\kappa)$. For $q = q^{(2)}$ the field is pure magnetic one: $E_1 = E_2 = E_3 = 0,\ H_1 = H_3 = 0,\ H_2 = \mathcal{F}_{13}$. For $q = q^{(3)}$ there are magnetic and electric fields: $E_1 = E_2 = H_1 = H_3 = 0$, $E_3 = H_2 = \mathcal{F}_{13}$.

To present the obtained results for all three cases in a compact form we introduce the following notations: $\mathrm{F}_1 \equiv E_3$ for $q = q^{(1)}$, $\mathrm{F}_2 \equiv H_2$ for $q = q^{(1)}$, and $\mathrm{F}_3 \equiv E_3$ for $q = q^{(3)}$. Then near the plane the strengths of fields are

$$\mathrm{F}_i(x_3) \underset{x_3\to 0}{=} \frac{f_i}{m^2x_3^2} + \frac{g_i}{m|x_3|} + \frac{(-1)^{i+1}e\chi\kappa}{2} - 2f_i + O(x_3),$$

where,

$$f_1 = \frac{em^2}{8\pi^2\alpha_1^2}\left[(1+\alpha_1^2)\mathrm{Arctg}(\alpha_1) - \alpha_1\right],\ g_1 = \frac{e\beta_1 m^2}{4\pi^2\alpha_1^2}\left[\mathrm{Arctg}(\alpha_1) - \alpha_1\right],$$

$$f_2 = \frac{em^2}{16\pi^2\alpha_2^2}\left[(1-\alpha_2^2)\ln\frac{1+\alpha_2}{1-\alpha_2} - 2\alpha_2\right],\ g_2 = \frac{e\beta_2 m^2}{8\pi^2\alpha_2^2}\left[\ln\frac{1+\alpha_2}{1-\alpha_2} - 2\alpha_2\right].$$

$$f_3 = \frac{e\kappa m^2}{3\pi^2(4+\tau^2+\lambda^2)},\ g_3 = -\frac{4e\kappa\lambda m^2}{3\pi^2(4+\tau^2+\lambda^2)^2}.$$

Here we used the notations:

$$\alpha_1 = \frac{4\kappa}{4-\kappa^2+\tau^2+\lambda^2},\ \alpha_2 = \frac{4\kappa}{4+\kappa^2+\tau^2+\lambda^2},\quad \beta_i = \frac{\lambda\alpha_i}{\kappa},\ i = 1,2.$$

For the large distance asymptotic we have the following results:

$$\mathrm{F}_i(x_3) \underset{x_3\to\infty}{=} u_i + \frac{(-1)^{i+1}e\chi\kappa}{2} - 2f_i + \frac{v_i e^{-2m|x_3|}}{(4m\pi|x_3|)^{3/2}}\left\{1 + O\left(1/x_3\right)\right\}$$

The constants u_i,$v_i$for $i = 1, 2$ are written as

$$u_i = \frac{em^2}{4\pi^2\alpha_i^2}\left\{\frac{\alpha_i{\beta_i}^2}{1-(-1)^i{\alpha_i}^2} + \sum_{k=1}^{2}\frac{b_k^{(i)}\ln\left[-a_k^{(i)} - \sqrt{(a_k^{(i)})^2-1}\right]}{\sqrt{(a_k^{(i)})^2-1}}\right\},\ v_i = \frac{e\alpha_i m^2}{1+\beta_i},$$

where

$$a_1^{(1)} = -\frac{\beta_1 + i\alpha_1\sqrt{{\beta_1}^2-{\alpha_1}^2-1}}{1+{\alpha_1}^2},\ b_1^{(1)} = \frac{({\beta_1}^2-{\alpha_1}^2)(\alpha_1\beta_1 - i\sqrt{{\beta_1}^2-{\alpha_1}^2-1})}{2(1+{\alpha_1}^2)},$$

$$a_2^{(1)} = -\frac{\beta_1 - i\alpha_1\sqrt{{\beta_1}^2-{\alpha_1}^2-1}}{1+{\alpha_1}^2},\ b_2^{(1)} = \frac{({\beta_1}^2-{\alpha_1}^2)(\alpha_1\beta_1 + i\sqrt{{\beta_1}^2-{\alpha_1}^2-1})}{2(1+{\alpha_1}^2)},$$

$$a_1^{(2)} = -\frac{\beta_2 + \alpha_2\sqrt{{\alpha_2}^2+{\beta_2}^2-1}}{1-{\alpha_2}^2},\ b_1^{(2)} = \frac{({\alpha_2}^2+{\beta_2}^2)(\alpha_2\beta_2 + \sqrt{{\alpha_2}^2+{\beta_2}^2-1})}{2(1-{\alpha_2}^2)},$$

$$a_2^{(2)} = -\frac{\beta_2 - \alpha_2\sqrt{{\alpha_2}^2+{\beta_2}^2-1}}{1-{\alpha_2}^2},\ b_2^{(2)} = \frac{({\alpha_2}^2+{\beta_2}^2)(\alpha_2\beta_2 - \sqrt{{\alpha_2}^2+{\beta_2}^2-1})}{2(1-{\alpha_2}^2)}.$$

For u_3, v_3 one obtains the following expressions

$$u_3 = \frac{e\kappa m^2}{3\pi^2(4+\tau^2+\lambda^2)}\left(3 - \frac{32\lambda^2}{(4+\tau^2+\lambda^2)^2}\right),\ v_3 = \frac{4e\kappa}{\tau^2+(2+\lambda)^2}.$$

5 Fields generated by simplest defects

The obtained results demonstrate the non-trivial dependence of the field E and H on the parameters κ, λ, τ. In the main approximation E and H are linear functions of χ, and $E = H = 0$, if $\chi = 0$, $\kappa = 0$. Let us consider the simplest non-trivial case $\kappa \neq 0$, $\chi = \lambda = \tau = 0$.

Asymptotics of E and H for large and small x_3 are the following. If $q = q^{(1)}$, the defect generates pure electric field E_3

$$E_3 \underset{x_3\to 0}{\approx} \frac{em^2}{8\pi^2\omega^2}\left[(1+\omega^2)\mathrm{Arctg}(\omega) - \omega\right]\left(\frac{1}{m^2x_3^2} - 2\right). \tag{18}$$

$$E_3 \underset{x_3\to\infty}{\approx} -\frac{em^2}{4\pi^2\omega^2}\left[\mathrm{Arctg}(\omega) - \omega\right],\ \omega = \frac{4\kappa}{4-\kappa^2}.$$

For $q = q^{(2)}$ the field is pure magnetic

$$H_2 \underset{x_3\to\ 0}{\approx} \frac{em^2}{16\pi^2\omega'^2}\left[(1+\omega'^2)\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right]\left(\frac{1}{m^2x_3^2} - 2\right). \tag{19}$$

$$H_2 \underset{x_3 \to \infty}{\approx} -\frac{em^2}{8\pi^2\omega^{2'}}\left[\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right],\ \omega' = \frac{4\kappa}{4+\kappa^2}.$$

For $q = q^{(3)}$, $E_1 = E_2 = H_1 = H_3 = 0$, and asymptotics of the fields E_3, H_2 are of the form

$$E_3 \underset{x_3 \to\ 0}{\approx} H_2 \underset{x_3 \to\ 0}{\approx} \frac{e\kappa m^2}{12\pi^2}\left(\frac{1}{m^2 x_3^2} - 2\right), \tag{20}$$

$$E_3 \underset{x_3 \to\ \infty}{\approx} H_2 \underset{x_3 \to\ \infty}{\approx} \frac{e\kappa m^2}{12\pi^2}.$$

In all cases the fields are constant at large distances from the plane. It is in agreement with well known results for homogeneous charge and current distributions on the plane in the classical electrodynamics [30, 31]. The magnitudes of the fields are defined by the factors $m^2/16\pi^2$, $m^2/8\pi^2$, $m^2/4\pi^2$. They are of the order $10^{17}\ cm^{-2} \div 4\cdot 10^{17}\ cm^{-2}$. It is more than the typical surface density $10^{16}\ cm^{-2}$ of atoms on the boundary of solids. The field $E(x_3)$ is a monotonic function of x_3. Therefore comparing the large- and small-distance asymptotics (18), (5), we can obtain an estimation for maximal distance d_m from the plane for which (18) can be used:

$$\frac{1}{m^2 d_m^2} \geq 2 + c(\omega), \quad c(\omega) \equiv 2\frac{\mathrm{Arctg}(\omega) - \omega}{\omega - (1+\omega^2)\mathrm{Arctg}(\omega)}.$$

Since $0 < c(\omega) \leq 1$, we have $d_m \sim 1,4\cdot 10^{-10}cm \div 1,7\cdot 10^{-10}cm$. The ranges of validity $x_3 < d_m$ of asimptotics (19), (20) can be found analogously:
$d_m \sim 2,0\cdot 10^{-10}cm \div 2,4\cdot 10^{-10}cm$ for (19), and $d_m \sim 1,7\cdot 10^{-10}cm$ for (20).

Important feature of the fields generated by the considered defects for $q = q^{(1)}, q^{(2)}$ is that they are singular by $\kappa = \pm 2$. It means that these values of parameter κ are the phase transition points, where in the case $q = q^{(1)}$ the electrical field E_3 is changed in a sudden way, and in the case $q = q^{(2)}$ the field H_2 become infinite. This phenomena seems to be not very surprising since it is similar to the known supercritical effects induced by perturbation of Dirac field by attractive δ potential which causes diving of the ground state into the Dirac see by finite value of the coupling parameter [33, 34, 35].

The points $\kappa = \pm 2$ are stable in respect to transformation $\kappa \to \kappa' = 4/\kappa$, for which $\omega(\kappa) \to \omega(\kappa') = -\omega(\kappa)$, $E_3(\kappa) \to E_3(\kappa') = -E_3(\kappa)$, and $\omega'(\kappa) \to \omega'(\kappa') = \omega'(\kappa)$, $H_2(\kappa) \to H_2(\kappa') = H_2(\kappa)$. For $q = q^{(1)}$ in each point of space the magnitude of field E_3, considered as a functions of κ is restricted: $|E(\kappa)| \leq |E(2)| < \infty$ for all values of κ.

For the $\lambda = \chi = \tau = 0$ the short distance asymptotic is of the form $E, H \sim const[1/(m^2x^2) - 2]$. In this case the relative correction of the next to leading term appears to be independent on parameter κ describing specific properties of the defect on the plane.

6 Conclusions

Suggested model describes an infinite plane with homogeneous charge and current distributions in the framework of QED. Specific properties of the physical system are characterised by additional term S_{def} (action of defect) combined with the usual action of QED into the full action of the model. S_{def} was chosen on the basis of general principles of QED: locality, gauge invariance and renormalizability of the theory. It contains 4 parameters which can be fixed by normalisation conditions.

The calculation of the leading order effects shows that mean field induced by S_{def} has classical behaviour at large distance from the plane. Corresponding asymptotic can be used as normalisation conditions pinpointing interplay of parameters describing the fields of a plane in classical ED [30, 31] with ones of considered model. Along this way one can express four parameters of S_{def} in terms of the effective charge and current densities and constants characterising macroscopic properties of material of the plane.

On short distances the fields E, H are singular as functions of distance x from the defect : $E, H \sim \text{const}/x^2$. Estimating energy density with usual classical formula $W = (E^2 + H^2)/2$, one obtains its behaviour for (18),(19) and (20) at short distances as $W(x) \sim c/x^4 + c'/x^2$, where c, c' are finite independent from cut-off constants. This singularities representing physical peculiarities of the model could be predicted with dimensional analysis. It is similar with one found for scalar field under Dirichlet or Neumann boundary conditions on a single plate [36], akin local effects near surfaces can be observed in different geometries (see, for example, [37, 38, 39]). Our model predicts dependence of c and c' on parameters of the material plane.

Properties of material film described by S_{def} are defined by classical 4-current (given by $S_l(A)$) and particle density and 4-current of quantum of electron-positron field (given by $S_{\lambda q}(\bar{\psi}, \psi)$). Thus, it is assumed that the film consists of infinitely heavy charged particles, electrons and positrons uniformly filling the plane. Fluctuation of Dirac field in QED generates an effective width of the film. It is of the order $10^{-10}cm$. In more realistic model one should describe the heavy charges in the framework of quantum fields theory. Simplest model of this kind could be QED with additionally proton-antiproton field. Fluctuations of this field generate anomalous electromagnetic field at a distance of order $10^{-13}cm$ (because proton is 2000 times heavier then electron). Thus, they are insufficient for the effects at a distance of order $10^{-10}cm$ calculated in this paper. The leading singularity of the electromagnetic field near the plane is mass independent. Therefore in virtue of opposite charges of electron and proton, it is canceled for neutral defect with the same parameters $\lambda, \vec{q}$ for both fermion fields, and at the distances x less than $10^{-13}cm$ their fluctuation generates the fields of the form $< E > \sim (M - m)C_e/|x| + (M^2 - m^2)C'_e$, $< H > \sim (M - m)C_h/|x| + (M^2 - m^2)C'_h$, where M (m) is the mass of proton (electron), C_e, C'_e, C_h, C'_h are functions of parameters $\lambda, \vec{q}$, and $C_e = C_h = 0$ if $\lambda = 0$.

In the model of homogeneous defect the the surface energy density is infinite because of ultra-violet divergences, and needs a regularization with cut-off impulse Λ forming

effectively a discreet lattice structure with characteristic scale $\sim \Lambda^{-1}$ in the coordinate space. It can be considered as a model of film consisting of atoms. Analysis of the function $F(x_3, \Lambda)$ used for calculations of the fields $\mathcal{F}_{\mu\nu}(x_3)$ show that fields near the plane for discreet defect $\mathcal{F}_{\mu\nu}(x_3)$ can be described if we replace in results for continuous defect the singularities of $\mathcal{F}_{\mu\nu}(x_3)$ generated by fermion field of mass m_f as follows: $1/x_3^2 \to f_2(x_3, \mu) = (1-(1+\mu x)\exp\{-\mu|x_3|\})/x_3^2$, $1/|x_3| \to f_1(x_3,\mu) = (1-\exp\{-\mu|x_3|\})/|x_3|$ where $\mu = \min\{\Lambda, m_f\}$. Such approximations are valid for distances smaller as $\sim 1/\mu$. Thus, for neutral discreet defect with $\Lambda^{-1} \sim 10^{-8} - 10^{-7} cm$ the behaviour of the fields $\mathcal{F}_{\mu\nu}(x_3)$ on the distances $\sim \Lambda^{-1}$ can be approximately described by main terms of short distance asymptotic of the homogeneous model. It is not surprising since there is no sharp cutoff dependence in the model of renormalizable quantum field theory.

In our paper we have restricted ourself with the simple problem to calculate the mean electromagnetic field generated by perturbation of QED vacuum by infinite plane film. It is important to note that a finite physical observable is extracted for a system of just one isolated plane in distinction to ordinary Casimir effect. Obtained formulas predict dependence of Casimir-Polder forces on charge and current densities which can be proved experimentally. In the the critical point $\kappa = \pm 2$ the magnetic field becomes infinite, and in the vicinity of critical point it must be observable near the defect surface with overcoming exponential suppression from masses of fermions. Other way to test the proposed model is the usage of obtained results for calculation of some physical feature of the film (for example, the spectrum of bound states of charged particle in anomalous electric potential of neutral defect) which can be investigated experimentally. In the framework of the model one can study scattering of electrons and photons on the defect and obtain results for experimental verifying.

We hope that the proposed approach can provide a deeper insight into the nature of quantum phenomena of interaction of macroscopic bodies with QED fields.

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